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Simple Robust Control Laws for Robot Manipulators, Part II: Adaptive Case

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1. Abstract

A new class of asymptotically stable adaptive control laws is introduced for application to the robotic manipulator. Unlike most applications of adaptive control theory to robotic manipulators, this analysis addresses the nonlinear dynamics directly without approximation, linearization, or ad-hoc assumptions, and utilizes a parameterization based on physical (time-invariant) quantities. This approach is made possible by using energy-like Lyapunov functions which retain the nonlinear character and structure of the dynamics, rather than *simple* quadratic forms which are ubiquitous to the adaptive control literature, and which have bound the theory tightly to linear systems with unknown parameters. It is a unique feature of these results that the adaptive forms arise by straightforward certainty equivalence adaptation of their nonadaptive counterparts found in the companion to this paper (i.e., by replacing unknown quantities by their estimates) and that this simple approach leads to asymptotically stable closed-loop adaptive systems. Furthermore, it is emphasized that this approach does not require convergence of the parameter estimates (i.e., via persistent excitation), invertibility of the mass matrix estimate, or measurement of the joint accelerations.

1. Introduction

In past years, many papers have appeared on the application of adaptive control theory to robotic manipulators (cf., [2]-[7], and Hsia [8] for overview). It is a general property of adaptive designs based on Lyapunov's Direct Method, that the Lyapunov function is chosen as a simple quadratic type, well-known and well studied in the standard adaptive control literature [12][13]. However, this particular Lyapunov function was originally motivated for applications to the standard adaptive control problems (i.e., linear systems with unknown parameters), and not for nonlinear dynamical systems. Hence, applications of standard adaptive control techniques to robotic manipulators invariably require the dynamics to be considered as linear. This in turn, requires the use of ad-hoc assumptions and/or analysis techniques including 1) treatment of *position dependent quantities as unknown constants, for which they must be assumed* to vary slowly with time; 2) linearization of the system about some local operating point-valid only for small excursions from nominal; 3) the use of linear decoupled models for the links, which neglects nonlinearities and crosscoupling effects; and 4) neglecting the nonlinear and time-varying dynamics completely by assuming the plant is linear. Hence, stability results based on these assumptions are questionable, and a rigorous proof of stability for adaptive control of robotic manipulators remains unresolved.

A recent exception to the above criticism is due to the work of Craig, Hsu and Sastry [9]. Here, a useful "linear in the parameters" formulation is exploited to simplify the analysis, and to demonstrate global convergence of an adaptive version of the computed-torque control law - without approximation to the nonlinear dynamics. However, the resulting adaptive controller requires the invertibility of the mass matrix estimate (which is not guaranteed a-priori), and measurement of the joint accelerations (which is generally unavailable). It is suggested in [9], that the former can be handled by projecting parameter estimates into known regions of parameter space for which the mass matrix inverse exists, and in which the true parameters are required to lie. However, knowledge and calculation of such regions is not straightforward and appears to be a weakness of the method.

In this paper, the "linear in parameters" formulation of [9] is used in conjunction with a different Lyapunov function. Here, the choice of Lyapunov function is more closely related to the energy of the system, and better retains the nonlinear structure and character of the dynamics. In addition, many problems associated with adapting the computed-torque control law directly are avoided by making use of the new class of exponentially stabilizing controllers introduced in [1]. Although these controllers are very similar in form to the computed torque method, they have many advantages in the nonadaptive case (cf., [1]), and have the unique property that they can be made adaptive by using a straightforward certainty equivalence approach (i.e., by replacing unknown quantities by their on-line estimates). Furthermore, the class of adaptive systems defined in this manner can be shown to be asymptotically stable without approximation to the nonlinear manipulator dynamics. This approach does not require convergence of parameter estimates (i.e., via persistent excitation), invertibility of the mass matrix estimate, or measurement of joint accelerations.

In the most recent literature (i.e., preprints, conference papers, etc.) there appears to be other work currently taking place which combines the linear in parameters formulation with a new Lyapunov function [10], [11]. Although this work is very new and is evolving very rapidly, we will try to contrast our results where possible, and provide an overall perspective.

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The format of the paper is as follows. In Sec. 2 the results of [1] are reviewed and summarized as required for treatment of the adaptive control case. In Sec. 3, asymptotic stability is proved for the class of systems arising from certainty equivalence adaptation of the control laws in [1]. Slightly tangential to the main thrust of the paper is the analysis in Sec. 4 of the adaptive computed torque method. Since the computed-torque control law is widely established in the literature, and widely applied in practice, it is useful to apply the techniques developed herein to see to what extent it can be made adaptive and to what extent stability can be guaranteed. In Sec. 5, several remarks are made pertinent to the new adaptive designs, and conclusions are given in Sec. 6.

2. Background and Notation

2.1 Manipulator Dynamics

The well-known Lagrange-Euler equations of motion for the n-joint manipulator is given as follows,

$$\dot{q}_1 = q_2 \tag{2.1}$$

$$M(q_1)\dot{q}_2 = - C(q_1,q_2) - k(q_1) + u \tag{2.2}$$

where

$$C(q_1,q_2) = \sum_{i=1}^{n} [(e_i q_2^T M_i(q_1))^T - \frac{1}{2} (e_i q_2^T M_i(q_1))] \tag{2.3}$$

$$e_i \triangleq i^{th} \text{ unit vector}$$

$$M_i(q_1) = \frac{\partial M(q_1)}{\partial q_{1i}} \quad ; \quad q_{1i} \triangleq i^{th} \text{ component of } q_1$$

$$k(q_1) \triangleq \text{gravity load}$$

Here, $u \epsilon R^n$ is a generalized torque vector, q_1, q_2, $\dot{q}_2 \epsilon R^n$ are generalized joint position, velocity and acceleration vector, (e.g., q_1 is an angle or a distance for a revolute or prismatic joint, respectively, $M(q_1) \epsilon R^{n \times n}$ is the symmetric positive definite mass inertia matrix; $C(q_1,q_2) \epsilon R^n$ is the Coriolis and centrifugal force vector; and $k(q_1) \epsilon R^n$ is the gravitational load vector.

2.2 Some Useful Identities

Let the following notations be defined,

$$M_D(q_1,z) = \sum_{i=1}^{n} M_i(q_1) z \, e_i^T$$

$$\dot{M}(q_1,q_2) = \frac{d}{dt} M(q_1) = \sum_{i=1}^{n} M_i(q_1) \, e_i^T q_2$$

$$J(q_1,z) = \sum_{i=1}^{n} [(e_i z^T M_i(q_1) - (e_i z^T M_i(q_1))^T]$$

$$\Delta q_1 \triangleq q_1 - q_{1d} \quad , \quad \Delta q_2 \triangleq q_2 - q_{2d}$$

q_{1d}, $q_{2d} \triangleq$ desired joint position and velocities respectively $(q_{2d} = \dot{q}_{1d})$

$$r(q_1,q_2,q_{2d}) = \Delta q_2^T [\frac{1}{2} \dot{M}(q_1,q_2)\Delta q_2 - C(q_1,q_2)q_2]$$

Using the above notation, the following identities are quoted from [1] without proof. In these identities, x, y and z are used to denote arbitrary vectors of appropriate dimension,

<u>Identity 1</u>

$$\dot{M}(q_1,q_2)z = M_D(q_1,z)q_2 \quad \text{where vector } z \text{ is arbitrary}$$

<u>Identity 2</u>

$$C(q_1,z)z = \frac{1}{2} (M_D(q_1,z) - J(q_1,z))z$$

<u>Identity 3</u>

$$J(q_1,z) = M_D^T(q_1,z) - M_D(q_1,z)$$

2.3 Important Lemma

In this section, a useful lemma is reviewed, quoted directly without proof from [1]. For convenience, this result will be alternatively referred to as the β-Ball Lemma due to the method used to prove it.

Lemma 2-1 (β-Ball Lemma)

Given a dynamical system

$$\dot{x}_i = f_i(x_1,\ldots x_N,t) \quad , \quad x_i \varepsilon R^{n_i} \quad , \quad t \geq 0$$

Let f_i's be locally Lipschitz with respect to $x_1,\ldots,x_N$ uniformly in t on bounded intervals and continuous in t for $t \geq 0$. Suppose a function $V: R^{n_1 \times \cdots \times n_N} \times R_+ \rightarrow R_+$ is given such that

$$V(x_1,\ldots,x_N,t) = \sum_{i,j=1}^{N} x_i^T P_{ij}(x_1,\ldots,x_j,t)x_j \quad ,$$

V is positive definite in $x_1,\ldots,x_N$

$$\dot{V}(x_1,\ldots,x_N,t) \leq - \sum_{i\varepsilon I_1} (\alpha_i - \sum_{j\varepsilon I_{2i}} \gamma_{ij}||x_j(t)||^{k_{ij}})||x_i(t)||^2 \tag{2.4}$$

where α_i, γ_{ij}, $k_{ij} > 0$, $I_{2i} \subset I_1 \subset \{1,\ldots,N\}$

Let $\xi_i > 0$ be such that,

$$\xi_i||x_i||^2 \leq V(x_1,\ldots,x_N,t) \tag{2.5}$$

Let $V_o \triangleq V(x_1(0),\ldots,x_N(0),0)$

If $\forall i\varepsilon I_1$,

$$\alpha_i > \sum_{j\varepsilon I_{2i}} \gamma_{ij}\left(\frac{V_o}{\xi_j}\right)^{\frac{k_{ij}}{2}} \tag{2.6}$$

then $\forall \; \lambda_i \varepsilon (0, \; \alpha_i - \sum_{j\varepsilon I_{2i}} \gamma_{ij}\left(\frac{V_o}{\xi_j}\right)^{\frac{k_{ij}}{2}})$,

$$\dot{V}(x_1,\ldots,x_N,t) \leq - \sum_{i\varepsilon I_1} \lambda_i \; ||x_i||^2 \quad \forall \; t \geq 0 \qquad \blacksquare$$

2.4 Exponentially Stabilizing Control Laws

In [1], various new exponentially stabilizing compensators were introduced for both the set-point and tracking control problems. For the purposes of adaptive control, it is of interest to consider the subset of this class summarized in Table I. In addition, the well-known computed torque control has also been included in Table I for comparison purposes. It is noted that the desired potential field $U^*(\Delta q_1)$ used in [1] has been chosen here simply as,

$$U^*(\Delta q_1) = \frac{1}{2} \Delta q_1^T K_p \Delta q_1 , \tag{2.7}$$

so as not to obscure the presentation with additional obstacle avoidance objectives. Nevertheless, many of the adaptive control results presented herein are easily extended to the more general case.

It is useful to observe that all Control Laws 1-7 differ from the computed torque method in that the mass matrix $M(q_1)$ does not premultiply the position and velocity feedback gains K_p and K_v respectively. This property is critical since it renders this entire class of control laws amenable to simple adaptation schemes (i.e., certainty equivalence adaptation) which can be shown to lead to desired asymptotic stability properties. The presence of the mass matrix premultiplier otherwise prevents simple cancellations in the Lyapunov function derivative, hindering most attempts to apply adaptive control directly to the nonlinear dynamic manipulator equations. A recent exception to this can be found in the work of Craig, Hsu and Sastry [9]. However, the resulting adaptation law requires that the estimated mass matrix be invertible for all values of estimated parameters. This in turn requires on-line projections of parameter estimates into prespecified bounded regions of parameter space where $\hat{M}(q_1)$ is not only invertible, but where the true parameters are certain to lie. This approach not only requires tight bounds on parameter uncertainty, but involves a very difficult (al beit off-line) determination of the proper parameter projection domains. This problem is further exacerbated by the fact that the adaptation law is not parameterized by physical parameters and is of the form where the transformation back to physical parameters is neither straightforward or unique. These problems are overcome in this paper by using the exponentially stabilizing control laws of Table I, which do not involve a premultiplying mass matrix on the feedback gains.

TABLE I

COMPUTED TORQUE CONTROL LAW	CONDITIONS FOR STABILITY†	CROSS-REFERENCE TO [1]* AND [20]
$u = -M(q_1)(K_p\Delta q_1 + K_v\Delta q_2) + k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_2)q_2$	None required	(2.34)
NEW EXPONENTIALLY STABLE CONTROL LAWS		
1. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_2)q_{2d} + \frac{1}{2}M_D(q_1,q_{2d})q_2$	None required	(4.2a)
2. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_{2d})q_2 + \frac{1}{2}M_D(q_1,q_2)q_{2d}$	None Required	(4.2b)
3. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_{2d})q_{2d} + \frac{1}{2}M_D(q_1,q_2)q_{2d}$	None Required	(4.2c)
4. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_2)q_2 + \frac{1}{2}M_D(q_1,q_{2d})q_2$	None Required	(4.2d)
5. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_{2d})q_{2d}$	$\sigma_{min}(K_v) > \frac{n_2}{2}$	(4.6)
6. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_2)q_2$	$\sigma_{min}(K_v)$ sufficiently large w.r.t. Initial condition	(4.7)
7. $u = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_{1d}) + M(q_{1d})\dot{q}_{2d} + C(q_{1d},q_{2d})q_{2d}$	$\sigma_{min}(K_v)$ sufficiently large w.r.t. Initial condition	(4.8)

†General Assumptions $||q_{1d}||$, $||q_{2d}||$, $||\dot{q}_{2d}||$ bounded; $k_v = K_v^T > 0$, $K_p = K_p^T > 0$

*Let $U^*(\Delta q_1) \triangleq \frac{1}{2}\Delta q_1^T K_p \Delta q_1$ in [1]

In the nonadaptive case, comparisons between the new control laws of Table I and the computed torque method can be found in [1]. Nevertheless, a brief account is in order here. In particular, Control Laws 1, 2,3,4 are roughly "on par" with the computed torque method in the nonadaptive case, guaranteeing exponential stability with no conditions on K_p or K_v. Unlike the computed torque method, however, they are not in a form suitable for application of the recursive Newton-Euler computation technique. This presently appears to be their major disadvantage. In order to overcome this difficulty, Control Laws 5, 6 and 7 were developed in a form suitable for recursive Newton-Euler computation. Relative to the computed torque method, Control Law 5 utilizes the desired velocity signal q_{2d} in place of the measured velocity q_2 in the nonlinear terms of the controller. This "cleans up" the feedback signal in the sense that nonidealities due to sensor dynamics and measurement noise in q_2 are avoided in the nonlinear feedback terms. Control Law 7 further replaces q_1 in K, M and C by q_{1d}. This decouples the nonlinear terms from real-time measurements, which completely removes the requirement for on-line computation of nonlinear terms in the controller implementation. Control Law 6 is exactly the computed torque method without the premultiplying mass matrix term described earlier. The advantages of these controllers are off-set slightly by the conditions imposed on K_p and K_v for guaranteeing asymptotic stability i.e., that K_v be chosen sufficiently large for Control Laws 1, 2, 3, 4, 5, 6 and that both K_v and K_p be chosen sufficiently large for Control Law 7. It will be seen in the adaptive case that these requirements can be removed by adapting these feedback gains appropriately.

The use of q_{2d} rather than q_2 in many of the new control laws offers additional advantages. In particular, in the set-point control application $q_{2d}=\dot{q}_{2d}=0$. Hence, there is considerable simplification in the control laws relative to the computed torque method, i.e., the nonlinear terms vanish from the control law. This simplification carries over directly to the adaptive case and provides substantial simplification in set-point control relative to the recent adaptive control laws of Slotine and Li [11] and Paden [10].

3. A New Class of Asymptotically Stable Adaptive Control Laws

All of the new exponentially stabilizing control laws summarized in Table I have the unique property that can be adapted in real-time so as to yield asymptotically stable adaptive control systems. Furthermore, the adaptation is done in a certainty equivalence fashion, i.e., by simply replacing unknown quantities in the control laws by their estimates - as generated by an appropriate parameter adaptation algorithm. In this section, asymptotic stability for the various control laws will be proved, and the proper mechanisms for parameter adaptation will be derived.

The simplicity in structure of the adaptive control schemes presented here is largely due to a "linear in the parameters" formulation of the problem. This particular parameterization is becoming increasingly popular in recent literature (cf., [9][10][18][19]) and will be discussed in more detail below.

3.1 Linear in the Parameters Formulation

A useful parameterization of the nonlinear dynamical equations arises by noting the following relations (x, y and z arbitrary vectors),

$$C(x,y,)y = H_C(x,y)\theta_C$$

$$M(x)z = H_M(x,z)\theta_M$$

$$k(x) = H_k(x)\theta_k$$

$$M_D(x,y)y = H_D(x,y)\theta_D$$

where H_C, H_M, H_k and H_D are known matrix valued functions of x, y and z, and where θ_C, θ_M, θ_k, and θ_D are vectors of constant parameters related directly to true physical parameters (masses, inertias, link lengths, center of gravities, etc.). It is emphasized that this parameterization does not contain any hidden "slowly varying" states in the parameter vector definition and does not require any linearization of the dynamical equations of motion.

3.2 Global Asymptotic Stability for Adaptation of Control Laws 1, 2, 3, 4

In this section, global asymptotic stability is proved for adaptation of Control Laws 1, 2, 3 and 4. In order to avoid redundant analysis, the details of the proof will be considered only for Control Law 1, and the extension to the other control laws will follow immediately by taking advantage of the unified treatment of these control laws given in [1].

3.2.1 Asymptotic Stability

Consider Control Law 1,

$$u^o = -K_p\Delta q_1 - K_v\Delta q_2 + k(q_1) + M(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_2)q_{2d} + \frac{1}{2}M_D(q_1,q_{2d})q_2 \tag{3.1}$$

Here, superscript "o" is used to denote the ideal nonadaptive control law, i.e., the completely "tuned" control law which would be used if the parameters were known exactly. Using the linear in the parameters formulation discussed in Sec. 3.1 there exists a matrix $H_1(q_1, q_2, q_{2d}, \dot{q}_{2d})$ and a vector of parameters θ such that,

$$u^o = -K_p\Delta q_1 - K_v\Delta q_2 + H_1\theta \tag{3.2}$$

where

$$H_1\theta \triangleq M(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_2)q_{2d} + \frac{1}{2}M_D(q_1,q_{2d})q_2 \tag{3.3}$$

Here, the parameters in θ are constant with time and are related directly to physical link and payload parameters. When these parameters are unknown, the parameter vector θ is replaced by its estimate $\hat{\theta}(t)$ in real-time to give the following adaptive control law,

$$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_1\hat{\theta} \tag{3.4}$$

Subtracting (3.2) from (3.4) and rearranging gives

$$u = u^o + H_1(\hat{\theta}-\theta) \triangleq u^o + H_1\phi \tag{3.5}$$

This is an important relation since it shows that the adaptive control is equal to the nonadaptive control plus an expression which is linear in the parameter error $\phi \triangleq \hat{\theta}-\theta$.

The proof of stability then follows by choosing the following Lyapunov function,

$$V = V^o + \frac{1}{2}\phi^T\Gamma\phi \qquad \Gamma = \Gamma^T > 0 \tag{3.6a}$$

where V^o is the Lyapunov function for the nonadaptive control law used in [1], and where $\phi^T\Gamma\phi$ is a positive definite function in the parameter error ϕ. For completeness, V^o is rewritten here (cf., [1],(4.4) where $U^*(\Delta q_1) \triangleq \frac{1}{2}\Delta q_1^T K_p \Delta q_1$),

$$V^o = \frac{1}{2}\Delta q_2^T M(q_1)\Delta q_2 + \frac{1}{2}\Delta q_1^T (K_p + cK_v)\Delta q_1 + c\Delta q_1^T M(q_1)\Delta q_2 \qquad (3.6b)$$

Taking the derivative of V along system trajectories and substituting control law (3.5) gives upon rearranging,

$$\dot{V} = \dot{V}^o + (\Delta q_2 + c\Delta q_1)^T H_1\phi + \dot{\phi}^T\Gamma\phi \qquad (3.7)$$

where $\dot{V}^o$ is the Lyapunov function derivative for the nonadaptive case, and where the additional terms involving ϕ on the right hand side of (3.7) arise directly from the additional terms involving ϕ in the control law (3.5) and the Lyapunov function (3.6) respectively.

The second and third terms of (3.7) are cancelled exactly by the choice of adaptation law,

$$\dot{\phi} = \dot{\hat{\theta}} = -\Gamma^{-1}H_1^T(\Delta q_2 + c\Delta q_1) \qquad (3.8)$$

The expression for the remaining term $\dot{V}^o$ is simply taken from [1] as, (see [1], (4.5) where $\nu \triangleq \sigma_{min}(K_p)$, also note that Control Law 1 corresponds to case (4.2b) for which $a = \frac{3}{2}$)

$$\dot{V} = \dot{V}^o$$

$$= -\alpha_1||\Delta q_1||^2 - \alpha_2||\Delta q_2||^2 + \gamma_{21}\,||\Delta q_1||\,||\Delta q_2||^2 \qquad (3.9)$$

where

$$\alpha_1 = c(\sigma_{min}(K_p) - \frac{3}{4}\eta_2\rho^2) \qquad (3.10a)$$

$$\alpha_2 = \sigma_{min}(K_v) - c(\mu + \frac{3}{4}\frac{\eta_2}{\rho^2}) \qquad (3.10b)$$

$$\gamma_{21} = \frac{c}{2}\eta_1 \qquad (3.10c)$$

$$\eta_2 = \max_{q_{2d}} ||q_{2d}||\eta_1 \qquad (3.11a)$$

$$\eta_1 = \max_{q_1} (\sum_i ||M_i(q_1)||) \qquad (3.11b)$$

$$\mu = \max_{q_1} ||M(q_1)|| \qquad (3.11c)$$

$0 < c < \ell^2$ arbitrary

ρ^2 arbitrary

ℓ^2 arbitrary

Applying the β-ball argument of Lemma 2.1 to (3.9) using the values of α_1, α_2, and γ_{21} given in (3.10), it follows that if,

$$\sigma_{min}(K_p) > \frac{3}{4}\eta_2\rho^2 \qquad (3.12a)$$

$$\sigma_{min}(K_v) > c(\mu + \frac{3}{4}\frac{\eta_2}{\rho^2} + \frac{\eta_1}{2}(\frac{V_o}{\xi_1})^{\frac{1}{2}}) \qquad (3.12b)$$

Then,

$$\dot{V} \le -\lambda_1||\Delta q_1||^2 - \lambda_2||\Delta q_2||^2 \qquad (3.13)$$

for any λ_1 and λ_2 such that,

$$\lambda_1 \epsilon (0,\ c(\sigma_{min}(K_p) - \frac{3}{4}\eta_2\rho^2)) \qquad (3.14a)$$

$$\lambda_2 \epsilon (0,\ \sigma_{min}(K_v) - c(\mu + \frac{3}{4}\frac{\eta_2}{\rho^2} + \frac{\eta_1}{2}(\frac{V_o}{\xi_1})^{\frac{1}{2}}) \qquad (3.14b)$$

where

$$V_o = V|_{t=0} \qquad (3.15)$$

$$\xi_1 = \frac{1}{2}[\sigma_{min}(K_p + cK_v) - \ell^2 c\,\underline{\sigma}(M)] \qquad (3.16a)$$

$$\xi_2 = \frac{1}{2}(1 - \frac{c}{\ell^2})\,\underline{\sigma}(M) \qquad (3.16b)$$

$$\underline{\sigma}(M) \triangleq \min_{q_1} \sigma_{min}(M(q_1))$$

Since ρ^2 is arbitrary, it can be chosen sufficiently small so that (3.12a) is satisfied. With this choice of ρ^2, the value of c in (3.12b) can be chosen sufficiently small so that inequality (3.12b) is satisfied. Hence (3.13) follows. This is essentially the same proof of stability as in the nonadaptive case (c.f., [1] Theorem 4-1) with the following exceptions,

1) The value of V_o in (3.12b) and (3.14b) now includes the initial parameter error $\frac{1}{2}\phi^T(0)\Gamma\phi(0)$.

2) The value of c is now required for implementation of the parameter adaptation law (3.8).

3) $\dot{V}$ in (3.13) is now only negative semidefinite in the state since the full state vector in the adaptive case is augmented by ϕ.

It is noted that property 3 destroys the simple exponential stability argument used earlier in the nonadaptive case (cf., [1], Theorem 4-1) to insure asymptotic convergence of $||\Delta q_1||$ and $||\Delta q_2||$. In addition since the error system in $(\Delta q_1, \Delta q_2, \phi)$ is nonautonomous (and in general, nonperiodic), standard invariance principles are not applicable. Alternatively, we make use of a lemma due originally to Barbalat, quoted without proof from Popov [14] (pg. 211).

Lemma 3-1 (Barbalat)

If W is a real function of the real variable t, defined and uniformly continuous for $t \geq 0$ and if the limit of the integral

$$\lim_{t\to\infty} \int_0^t W(t')dt' ,$$

exists and is a finite number, then

$$\lim_{t\to\infty} W(t) = 0 . \qquad \blacksquare$$

For our purposes let,

$$W(t) \triangleq \lambda_1 ||\Delta q_1(t)||^2 + \lambda_2 ||\Delta q_2(t)||^2$$

so that

$$\dot{V} \leq - W \tag{3.17}$$

Integrating both sides of (3.17) from 0 to t, yields upon rearranging,

$$\int_0^t W\,dt' \leq V_o - V(t) \tag{3.18}$$

Since V_o is bounded, and V(t) is nonincreasing and bounded below, it follows that

$$\lim_{t\to\infty} \int_0^t W\,dt' < \infty$$

Also, since $\dot{W}$ is bounded, W(t) is uniformly continuous. Hence, application of Barbalat's Lemma gives,

$$\lim_{t\to\infty} W = 0 \tag{3.19}$$

or equivalently $||\Delta q_1||\to 0$ and $||\Delta q_2||\to 0$.

This completes the proof of asymptotic stability. The proof, however, is not a global one due to property 2, i.e., the value of c which was not required in the nonadaptive case now appears in the parameter adaptation law (3.8). Hence, one is committed to choosing a particular value of c in the adaptive implementation. Of course, c can always be chosen sufficiently small to satisfy the requirement, however, the position tracking performance determined by the magnitude of λ_1 in (3.14a) must be compromised as a result. Hence in practice, the initial choice of c can be made using whatever bounds on η_1, η_2, μ, $\underline{\sigma}(M)$ and V_o are available a-priori, and the value of c can be improved (increased) on-line as more information becomes available. It is noted that (3.16a) and (3.16b) impose additional constraints on how large c can become, since it is required that $\xi_1 > 0$ and $\xi_2 > 0$ for a positive definite V (these conditions can be shown sufficient).

The asymptotic stability proof presented above for adaptation of Control Law 1, is easily extended to adaptation of Control Laws 2, 3 and 4, since the corresponding nonadaptive Lyapunov function derivatives for these control laws are of exactly the same form as $\dot{V}^o$ in (3.25) (see [1], Theorem 4-1 for details). For convenience, all asymptotically stable adaptive control laws discussed thus far, and their appropriate parameter adaptation laws are summarized in Table II, corresponding to cases 1.a, 2.a, 3.a, and 4.a, respectively.

An alternative to choosing c sufficiently small in the above asymptotic stability argument is to choose K_v sufficiently large. In this case, the condition on c above can be removed completely by adapting K_v on-line. This modification insures global asymptotic stability of the adaptive control system (i.e., choice of c independent of the initial condition V_o) and is discussed in more detail below.

3.2.2 Global Asymptotic Stability-Adapting K_v

Since the velocity gain K_v enters linearly in the control law, it can be adapted as if it were an unknown parameter using the same formulation of Sec. 3.2.1. It will be shown that this approach removes the dependence of the choice of c on the initial condition V_o and this completes the proof of global asymptotic stability for the adaptive case.

Consider Control Law 1 written in adaptive form, where both θ and K_v are adapted in real time i.e.,

$$u = -K_p \Delta q_1 - \hat{K}_v \Delta q_2 + H_1 \hat{\theta} \tag{3.20}$$

Here, $\hat{K}_v$ is a time-varying quantity which remains to be specified, and H_1 is as defined earlier in (3.3). The nonadaptive control law u^o in (3.1) is subtracted from (3.20) to give the following expression,

$$u = u^o - \Delta K_v \,\Delta q_2 + H_1 \phi \tag{3.21}$$

where $\Delta K_v \triangleq \hat{K}_v - K_v$ and $\phi = \hat{\theta} - \theta$.

The Lyapunov function for the stability analysis is given as

$$V = V^o + \frac{1}{2}\phi^T \Gamma \phi + \frac{1}{2}\delta \, TR(\Delta K_v^T \Delta K_v) \ , \quad \delta > 0 \ , \quad \Gamma = \Gamma^T > 0 \tag{3.22}$$

where a new term has been added relative to (3.6a), quadratic in the error ΔK_v. Taking the derivative of V along system trajectories and substituting control law (3.21) gives upon rearranging

$$\dot{V} = \dot{V}^o + (\Delta q_2 + c\Delta q_1)^T H_1 \phi + \dot{\phi}^T \Gamma \phi$$

$$+ TR([\delta \Delta \dot{K}_v^T - \Delta q_2 (\Delta q_2 + c\Delta q_1)^T] \Delta K_v) \tag{3.23}$$

The latter terms are cancelled exactly by the choice of parameter adaptation laws,

$$\dot{\phi} = \dot{\hat{\theta}} = -\Gamma^{-1} H_1^T (\Delta q_2 + c\Delta q_1) \tag{3.24a}$$

$$\Delta \dot{K}_v = \dot{\hat{K}}_v = + \frac{1}{\delta} (\Delta q_2 + c\Delta q_1) \Delta q_2^T \tag{3.24b}$$

The choice leaves $\dot{V}$ exactly of the form (3.9) i.e., applying the β-Ball Lemma 2.1,

$$\dot{V} = \dot{V}^o \le -\lambda_1 ||\Delta q_1||^2 - \lambda_2 ||\Delta q_2||^2 \tag{3.25}$$

if,

$$\sigma_{min}(K_p) > \frac{3}{4} n_2 \rho^2 \tag{3.26}$$

$$\sigma_{min}(K_v) > c\left(\mu + \frac{3}{4}\frac{n_2}{\rho^2} + \frac{n_1}{2}\left(\frac{V_o}{\xi_1}\right)^{\frac{1}{2}}\right) \tag{3.27}$$

In (3.26) and (3.27), all quantities are defined exactly as in (3.12a) and (3.12b) respectively, except for V_o which is presently the initial value of V in (3.22). Furthermore, the values of ξ_1 and ξ_2 are once again given as

$$\xi_1 = \frac{1}{2}\left[\sigma_{min}(K_p + cK_v) - \ell^2 c\bar{\sigma}(M)\right] \tag{3.28}$$

$$\xi_2 = \frac{1}{2}\left(1 - \frac{c}{\ell^2}\right)\underline{\sigma}(M) \tag{3.29}$$

An important observation is that,

$$V_o \sim O(||K_v||^2) \ , \quad ||K_v|| \to \infty \tag{3.30a}$$

$$\xi_1 \sim O(||K_v||) \ , \quad ||K_v|| \to \infty \tag{3.30b}$$

Hence, for any choice of $\delta>0$, $c>0$ and $K_p = K_p^T > 0$, there exist values of ρ^2, ℓ^2, and $K_v = K_v^T > 0$ (with $\sigma_{min}(K_v)$ sufficiently large) such that inequalities (3.26) and (3.27) are satisfied, and $\xi_1 > 0$, $\xi_2 > 0$ in (3.28) and (3.29), respectively. Global asymptotic stability of this adaptive control scheme then follows immediately by application of Barbalat's Lemma to the Lyapunov function derivative (3.25), as was done earlier in equations (3.17) through (3.19).

The global asymptotic stability of adaptive controllers based on Control Laws 2, 3 and 4 (where K_v is adapted on-line) follow from an identical argument, since $\dot{V}^o$ corresponding to the nonadaptive Lyapunov function derivatives for these control laws are of exactly the same form as $\dot{V}^o$ in this analysis (see [1], Theorem 4-1 for details). For convenience, these adaptive control laws involving adaptation of K_v are summarized in Table II, corresponding to cases 1.b, 2.b, 3.b, and 4.b, respectively.

3.3 Global Asymptotic Stability for Adaptation of Control Laws 5, 6 and 7

Global asymptotic stability for adaptation of Control Laws 5, 6 and 7 can be proved using exactly the same techniques as applied in Sec. 3.2. The only difference lies in slight variations in the nonadaptive Lyapunov function derivative $\dot{V}^o$ which arises in each adaptive control analysis

Due to space limitations, these proofs have been omitted, but the results are summarized in Table II corresponding to cases 5.a, 5.b, 6.a, 6.b, and 7.a, 7.b, respectively. Details can be found in [21], to which the equation numbers in Table II are referenced.

4. Adaptive Computed Torque Method

It was mentioned earlier that in the computed torque method (i.e., control law 8) the presence of the $M(q_1)$ term premultiplying the K_p and K_v gains complicates the Lyapunov analysis and hinders most simple attempts to make it adaptive. Nevertheless, the computed torque controller is a well-known control law in the literature and is widely applied in practice. Hence, it is useful to investigate under what conditions it can be made adaptive, and to what extent adaptive stability can be guaranteed. For this purpose, we consider a special case of the computed torque control law which has scalar gains k_p and k_v, i.e.,

$$u^o = -M(q_1)(k_p\Delta q_1 + k_v\Delta q_2) + k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_2)q_2 \tag{4.1}$$

This is written in adaptive form as,

$$u = u^o + H_8\phi \tag{4.2}$$

where $\phi = \hat{\theta} - \theta$, and the linear in the parameters part has been chosen as,

$$H_8\theta \stackrel{\Delta}{=} -M(q_1)(k_p\Delta q_1+k_v\Delta q_2) + k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1,q_2)q_2 \tag{4.3}$$

Let a Lyapunov function be defined as,

$$V = V^o + \frac{1}{2}\phi^T\Gamma\phi \quad , \quad \Gamma = \Gamma^T > 0 \tag{4.4a}$$

where

$$V = \frac{1}{2}\Delta q_2^T M(q_1)\Delta q_2 + \frac{1}{2}(k_p+ck_v)\Delta q_1^T M(q_1)\Delta q_1 + c\Delta q_1^T M(q_1)\Delta q_2 \tag{4.4b}$$

Then, the derivative of (4.4) along system trajectories induced by control (4.2) is given by

$$\dot{V} = \Delta q_2^T [-k_v M(q_1)\Delta q_2 + \frac{1}{2}M_D(q_1,\Delta q_2)q_2]$$

$$+ c\Delta q_1^T [-k_p M(q_1)\Delta q_1 + M_D(q_1,\Delta q_2)q_2 + M_D(q_1,\Delta q_1)q_2] \tag{4.5}$$

$$+ (\Delta q_2 + c\Delta q_1)^T H_8\phi + \dot{\phi}^T\Gamma\phi \tag{4.5}$$

Let,

$$\dot{\phi} = \dot{\hat{\theta}} = -\Gamma^{-1} H_8^T(\Delta q_2+c\Delta q_1) \tag{4.6}$$

Then,

$$\dot{V} \leq -\alpha_1||\Delta q_1||^2 - \alpha_2||\Delta q_2||^2 + \gamma_{12}||\Delta q_2||\ ||\Delta q_1||^2$$

$$+ (\gamma_{21}||\Delta q_1|| + \gamma_{22}\ ||\Delta q_2||)||\Delta q_2||^2 \tag{4.7}$$

where

$$\alpha_1 = c(k_p\ \underline{\sigma}(M) - \eta_2(1 + \rho^2)) \tag{4.8}$$

$$\alpha_2 = k_v\underline{\sigma}(M) - \frac{1}{2}\eta_1 - \frac{c\eta_2}{\rho^2} \tag{4.9}$$

$$\gamma_{12} = c\eta_1 \tag{4.10}$$

$$\gamma_{21} = c\eta_1 \quad ; \quad \gamma_{22} = \frac{1}{2}\eta_1 \tag{4.11}$$

Applying the β-Ball Lemma, it follows that,

$$\dot{V} \leq -\lambda_1 ||\Delta q_1||^2 - \lambda_2 ||\Delta q_2||^2 \tag{4.12}$$

if,

$$c k_p \underline{\sigma}(M) > \eta_2 (1 + \rho^2)) + c\, \eta_2 \left(\frac{V_o}{\xi_1}\right)^{\frac{1}{2}} \tag{4.13}$$

$$k_v \underline{\sigma}(M) > \frac{1}{2} \eta_1 + \frac{c\eta_2}{\rho^2} + \frac{1}{2} \eta_1 \left(\frac{V_o}{\xi_2}\right)^{\frac{1}{2}} + c\, \eta_1 \left(\frac{V_o}{\xi_1}\right)^{\frac{1}{2}} \tag{4.14}$$

$$\xi_1 = \frac{1}{2} [k_p + c k_v - c\ell^2] \underline{\sigma}(M), \tag{4.15}$$

$$\xi_2 = \frac{1}{2} (1 - \frac{c}{\ell^2}) \underline{\sigma}(M) \quad ; \ \ell^2 > 0 \text{ arbitrary} \tag{4.16}$$

It is noted that for any $c > 0$, both k_p and k_v can always be chosen sufficiently large so that $\xi_1 > 0$ and $\xi_2 > 0$ (for appropriate choice of $\ell^2 > 0$ in (4.15), (4.16)), and inequalities (4.13) and (4.14) are satisfied. Hence, the adaptive computed torque control law given by (4.2), (4.3) with parameter adaptation (4.6) is asymptotically stable when k_p and k_v are chosen sufficiently large.

Since k_p and k_v must be chosen sufficiently large with respect to the initial condition V_0 (c.f., (4.13), (4.14)) this proof of asymptotic stability is not global (i.e., for fixed k_p and k_v there will always exist some V_0 such that λ_1 and/or λ_2 are not positive). For this particular algorithm, it is presently not clear how to adapt k_p and k_v to insure global asymptotic stability since the control u in (4.1) is not linear in the parameters (θ, k_p, k_v).

5. Summary and Remarks

The adaptive control laws derived herein, along with the sufficient conditions for stability and appropriate parameter adaptation laws are summarized in Table II. Several remarks are in order at this point in the discussion.

Remark 5-1 All adaptive control laws in this paper were derived for the general tracking control law. However, significant simplification occurs in many of these designs for the special case of set-point control (i.e., $q_{2d} = \dot{q}_{2d} = 0$).

Remark 5-2 The adaptive robustness issue remains open. Certainly for parameter adaptation laws of the form given in Table II, there will be sensitivities to noise disturbances and unmodelled dynamics directly analogous to those which arise in the linear adaptive control case. It presently appears that many of the robustness techniques developed in the linear adaptive control literature will carry over to the nonlinear adaptive control application. This conjecture, however, remains to be investigated.

Remark 5-3 In the nonadaptive case, many of the control laws in Table II are in a form appropriate for application of the recursive Newton-Euler computational algorithm. However, the Newton-Euler algorithm requires knowledge of all physical parameters-more parameters than are actually needed to control the system adaptively and more than are actually adapted on-line in the vector θ of Table II. Hence, the transformation from $\hat{\theta}$ back to physical parameters is required in order to salvage use of the Newton-Euler algorithm in the adaptive case. However, the transformation is generally nonlinear and will not lead to a unique solution unless further constraints are imposed. One typical set of constraints arises when only the payload mass is unknown. In the more general adaptive case, it is useful to note that all linear in the parameters expressions can be implemented directly, since representations of the form $H\theta$ are assumed to be available in symbolic form.

Remark 5-4 The control laws of Table I were derived in [1] for the general desired potential energy function. This feature was dropped in the adaptive case in order to simplify the analysis. However, it appears that the adaptive control laws developed herein can be extended to the more general case and this line of research presently under investigation.

Remark 5-5 A brief comparison with the recent results Paden [10] and Slotine and Li [11] is useful. In [10] [11], adaptive control laws are derived by choosing u to cancel various terms in the Lyapunov function derivative, rather than overbounding them (via Lemma 2.1) as was done here. This approach has the advantage of providing global asymptotic convergence without adapting gains K_v and K_p. The control laws, however, are by necessity more complex than those designs considered here, and do not simplify in the set-point control case.

6. Conclusions

A new class of asymptotically stable adaptive control laws is defined by adapting the control laws of [1] in a certainty equivalence fashion. These algorithms are proved to be asymptotically stable without approximations, linearizations or ad-hoc assumptions concerning the nonlinear manipulator dynamics. Furthermore, the asymptotic convergence properties can be made global by appropriate adaptation of feedback gains. On-going research efforts are directed at adaptive robustness, computation, and obstacle avoidance problems.

TABLE II SUMMARY OF ASYMPTOTICALLY STABLE ADAPTIVE CONTROL LAWS

CASE #	ADAPTIVE CONTROL	LINEAR IN PARAMETERS EXPRESSION	PARAMETER ADAPTATION	STABILITY CONDITIONS[†]	REMARKS
1.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_1\hat{\theta}$	$H_1\theta = k(q_1)+H(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_2)q_{2d} + \frac{1}{2}M_D(q_1,q_{2d})q_2$	$\dot{\hat{\theta}} = -\Gamma^{-1}H_1^T(\Delta q_2+c\Delta q_2)$	$\sigma_{min}(K\ )$ sufficiently large <u>or</u> c sufficiently small (see (3.12))	*Asymptotic Stability
1.b	$u = -K_p\Delta q_1 - \hat{K}_v\Delta q_2 + H_1\hat{\theta}$	$H_1\theta$ = same as 1.a above	$\dot{\hat{\theta}} = -\Gamma^{-1}H_1^T(\Delta q_2+c\Delta q_1)$ $\dot{\hat{K}}_v = \frac{1}{\delta}(\Delta q_2+c\Delta q_1)\Delta q_2^T$	None required	*Global Asymptotic Stability
2.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_2\hat{\theta}$	$H_2\theta=k(q_1)+H(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_{2d})q_2 + \frac{1}{2}M_D(q_1,q_2)q_{2d}$	$\dot{\hat{\theta}} = -\Gamma^{-1}H_2^T(\Delta q_2+c\Delta q_1)$	$\sigma_{min}(K_v)$ sufficiently large <u>or</u> c sufficiently small	*Asymptotic Stability
2.b	$u = -K_p\Delta q_1 - \hat{K}_v\Delta q_2 + H_2\hat{\theta}$	$H_2\theta$ = same as 2.a above	$\dot{\hat{\theta}} = -\Gamma^{-1}H_2^T(\Delta q_2+c\Delta q_1)$ $\dot{\hat{K}}_v = -\frac{1}{\delta}(\Delta q_2+c\Delta q_1)\Delta q_2^T$	None required	*Global Asymptotic Stability
3.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_3\hat{\theta}$	$H_2\theta=k(q_1)+H(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_{2d})q_{2d} + \frac{1}{2}M_D(q_1,q_2)q_{2d}$	$\dot{\hat{\theta}} = -\Gamma^{-1}H_3^T(\Delta q_2+c\Delta q_1)$	$\sigma_{min}(K_v)$ sufficiently large <u>or</u> c sufficiently small	*Asymptotic Stability
3.b	$u = -K_p\Delta q_1 - \hat{K}_v\Delta q_2 + H_3\hat{\theta}$	$H_3\theta \triangleq$ same as 3.a above	$\dot{\hat{\theta}} = -\Gamma^{-1}H_3^T(\Delta q_2+c\Delta q_1)$ $\dot{\hat{K}}_v = -\frac{1}{\delta}(\Delta q_2+c\Delta q_1)\Delta q_2^T$	None required	*Global Asymptotic Stability
4.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_4\hat{\theta}$	$H_4\theta=k(q_1)+H(q_1)\dot{q}_{2d} - \frac{1}{2}J(q_1,q_2)q_2 + \frac{1}{2}M_D(q_1,q_{2d})q_2$	$\dot{\hat{\theta}} = -\Gamma^{-1}H_4^T(\Delta q_2+c\Delta q_1)$	$\sigma_{min}(K_v)$ sufficiently large <u>or</u> c sufficiently small	Asymptotic Stability

4.b	$u = -K_p\Delta q_1 - K\Delta q_2 + H_4\hat{\theta}$	$H_4\theta \overset{\Delta}{=}$ same as 4.a above	$\dot{\hat{\theta}} = -\Gamma^T H_4^T(\Delta q_2 + c\Delta q_1)$ $\dot{\hat{K}}_v = \frac{1}{\delta}(\Delta q_2 + c\Delta q_1)\Delta q_2^T$	None required	Global Asymptotic Stability
5.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_5\hat{\theta}$	$H_5 = K(q_1) + M(q_1)\dot{q}_{2d} + C(q_1, q_{2d})q_{2d}$	$\dot{\hat{\theta}} = -\Gamma^{-1} H_5^T(\Delta q_2 + c\Delta q_1)$	$\sigma_{min}(K_v)$ sufficiently large (see (3.36), (3.37))	*† Asymptotic Stability
5.b	$u = -K_p\Delta q_1 - \hat{K}_v\Delta q_2 + H_5\hat{\theta}$	$H_5\theta$ - same as 3.a above	$\dot{\hat{\theta}} = -\Gamma^{-1} H_5^T(\Delta q_2 + c\Delta q_1)$ $\dot{\hat{K}}_v = \frac{1}{\delta}(\Delta q_2 + c\Delta q_1)\Delta q_2^T$	None required	*† Global Asymptotic Stability
6.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_6\hat{\theta}$	$H_6\theta = k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1, q_2)q_2$	$\dot{\hat{\theta}} = -\Gamma^{-1} H_6^T(\Delta q_2 + c\Delta q_1)$	$\sigma_{min}(K_v)$ sufficiently large (see (3.45), (3.46))	† Asymptotic Stability
6.b	$u = -K_p\Delta q - \hat{K}_v\Delta q_2 + H_6\hat{\theta}$	$H_6\theta$ = same as 4.a above	$\dot{\hat{\theta}} = -\Gamma^{-1} H_6^T(\Delta q_2 + c\Delta q_1)$ $\dot{\hat{K}}_v = \frac{1}{\delta}(\Delta q_2 + c\Delta q_1)\Delta q_2^T$	δ sufficiently small (see (3.55), (3.56))	† Global Asymptotic Stability
7.a	$u = -K_p\Delta q_1 - K_v\Delta q_2 + H_7\hat{\theta}$	$H_7\theta = k(q_{1d}) + M(q_{1d})\dot{q}_{2d} + C(q_{1d}, q_{2d})q_{2d}$	$\dot{\hat{\theta}} = -\Gamma^{-1} H_7^T(\Delta q_2 + c\Delta q_1)$	$\sigma_{min}(K_p)$ and $\sigma_{min}(K_v)$ sufficiently large w.r.t. initial cond. (see (3.55), (3.56))	*† Asymptotic Stability
7.b	$u = -\hat{K}_p\Delta q_1 - \hat{K}_v\Delta q_2 + H_7\hat{\theta}$	$H_7\theta$ = same as 5.a above	$\dot{\hat{\theta}} = -\Gamma^{-1} H_7^T(\Delta q_2 + c\Delta q_1)$ $\dot{\hat{K}}_v = \frac{1}{\delta_v}(\Delta q_2 + c\Delta q_1)\Delta q_2^T,\ \delta_v > 0$ $\dot{\hat{K}}_p = \frac{1}{\delta_p}(\Delta q_2 + c\Delta q_1)\Delta q_1^T,\ \delta_p > 0$	None required	*† Global Asymptotic Stability
8.	$u = H_8\hat{\theta}$	$H_8\theta \overset{\Delta}{=} M(q_1)(k_p\Delta q_1 + k_v\Delta q_2) + k(q_1) + M(q_1)\dot{q}_{2d} + C(q_1, q_2)q_2$ $k_p > 0,\ k_v > 0$	$\dot{\hat{\theta}} = -\Gamma^{-1} H_8^T(\Delta q_2 + c\Delta q_1)$	k_p and K_v sufficiently large w.r.t. Initial Condition (see (4.13), (4.14))	† Computed Torque Method with scalar gains - Asymptotic Stability

ǂ General Assumptions:

$K_v = K_v^T > 0,\ K_p = K_p^T > 0,\ \Gamma = \Gamma^T > 0,\ c > 0,\ \delta > 0,\ \delta_p > 0,\ \delta_v > 0$

*Significant simplification for set-point control
$\dot{q}_{2d} = q_{2d} = 0$

†Recursive Newton Euler applicable in nonadaptive case

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References

[1] J.T. Wen and D.S. Bayard, "Simple Robust Control Laws for Robotic Manipulators - Part I: Nonadaptive Case," this Workshop.

[2] S. Dubowsky and D.T. Desforges, "The Application of Model-Referenced Adaptive Control to Robotic Manipulators," Trans. ASME J. Dynamic Systems, Measurement and Control, Vol. 101, 1979, pp. 193-200.

[3] A.J. Koivo and T.-H. Guo, "Adaptive Linear Controller for Robotic Manipulators," IEEE Trans. Auto. Contr., Vol. AC-28, No. 2, 1983, pp. 162-171.

[4] C.S.G. Lee and M.J. Chung, "An Adaptive Control Strategy for Mechanical Manipulators," IEEE Trans. Auto. Contr., Vol. AC-29, No. 9, 1984, pp. 837-840.

[5] M. Tomizuka and R. Horowitz, "Model Reference Adaptive Control of Mechanical Manipulators," IFAC Workshop on Adaptive Systems in Control and Signal Processing, San Francisco, CA, 1983.

[6] M. Takegaki and S. Arimoto, "An Adaptive Trajectory Control of Manipulators," Int. J. Contr., Vol. 34, No. 2, 1981, pp. 219-230.

[7] K.Y. Lim and M. Eslami, "Adaptive Controller Designs for Robotic Manipulator Systems Using Lyapunov Direct Method," IEEE Trans. Auto. Contr., Vol. AC-30, No. 12, 1985, pp. 1229-1233.

[8] T.C. Hsia, "Adaptive Control of Robot Manipulators - A Review," IEEE Int. Conf. Robotics and Automation, San Francisco, CA, 1986.

[9] J. Craig, P. Hsu, and S. Sastry, "Adaptive Control of Mechanical Manipulators," IEEE Conf. Robotics and Automation, San Francisco, CA, April 1986.

[10] B. Paden, "PD+ Robot Controllers: Tracking and Adaptive Control," Private Communication.

[11] J.-J.E. Slotine and W. Li, "On the Adaptive Control of Robot Manipulators," ASME Winter Meeting, Anaheim, CA, 1986.

[12] I.D. Landau, "A Survey of Model Reference Adaptive Techniques - Theory and Applications," Automatica, Vol. 11, 1974, pp. 353-379.

[13] K.S. Narendra and R.V. Monopoli, Eds., Applications of Adaptive Control, Academic Press, New York, 1980.

[14] V.M. Popov, Hyperstability of Control Systems, Springer-Verlag, New York, 1973.

[15] P.A. Ioannou and P.V. Kokotovic, "Robust Redesign of Adaptive Control," IEEE Trans. Auto. Contr., Vol. AC-29, No. 3, 1984, pp. 202-211.

[16] P.A. Ioannou, "Robust Adaptive Controller with Zero Residual Errors," IEEE Trans. Auto. Contr., Vol. AC-31, No. 8, 1986, pp. 773-776.

[17] D.S. Bayard, C.H.C. Ih and S.J. Wang, "Adaptive Control for Flexible Space Structures with Measurement Noise," submitted to American Control Conference, Minneapolis, Minnesota, June 10-12, 1987.

[18] C.G. Atkinson, C.G. An and J.M. Hollerbach, "Estimation of Initial Parameters of Manipulator Loads and Links," International Symposium on Robotics Research, 1985.

[19] P. Khosla and T. Kanade, "Parameter Identification of Robot Dynamics," IEEE Conf. Decision and Control, Fort Lauderdale, Florida, 1985.

[20] J.T. Wen and D.S. Bayard, Robust Control for Robotic Manipulators, Part I: Non-Adaptive Case, (JPL Internal Document, Engineering Memorandum, EM 347-87-203). Jet Propulsion Laboratory, Pasadena, California, 1987.

[21] D.S. Bayard and J.T. Wen, Robust Control for Robotic Manipulators, Part II: Adaptive Case, (JPL Internal Document, Engineering Memorandum, EM 347-87-204). Jet Propulsion Laboratory, Pasadena, California, 1987.